Gender Based Taxation and the Division of Family Chores *

Alberto Alesina Andrea Ichino
HARVARD UNIVERSITY UNIVERSITY OF BOLOGNA
Loukas Karabarbounis
HARVARD UNIVERSITY

October 3, 2007

Abstract

Gender Based Taxation (GBT) satisfies the Ramsey optimality criterion by taxing less the more elastic labor supply of (married) women. This holds when different elasticities between men and women are taken as exogenously and primitive. But in this paper we also explore differences in gender elasticities which emerge endogenously in a model in which spouses bargain over the allocation of home duties. GBT changes spouses’ implicit bargaining power and induces a more balanced allocation of house work and working opportunities between males and females. Because of decreasing returns to specialization in home and market work, social welfare improves by taxing conditional on gender. When income sharing within the family is substantial, both spouses may gain from GBT.

Keywords: Optimal Taxation, Economics of Gender, Family Economics, Elasticity of Labor Supply.

*We thank George Akerlof, Stefania Albanesi, George-Marios Angeletos, Claudia Goldin, Larry Katz, James Poterba, Emmanuel Saez, Ivan Werning and seminar participants at the Universities of Berkeley, Bologna, Jerusalem, MIT, Instituto Carlo Alberto in Torino, the CEPR-ESSLE and the NBER summer meeting, for helpful comments and suggestions.
1 Introduction

Optimal taxation theory prescribes that the government should tax less the goods and services which have a more elastic supply. Women labor supply is more elastic than men’s. Therefore, tax rates on labor income should be lower for women than for men.

This argument is well known in the literature, but it is not taken seriously as a policy proposal. This is surprising since a host of other gender based policies are routinely discussed, and often implemented, such as gender based affirmative action, quotas, different retirement policies for men and women, and also indirect gender based policies like child care subsidies, and maternal leaves.\footnote{For instance, gender based affirmative action is common in the US, Spain has recently introduced stringent quotas for female employment in many sectors and public support for child care is common in many European countries. Sweden has recently reformed paternal leave policies with the goal of inducing males to stay more at home with children and females to participate more continuously in the labor market.} Many of these gender based interventions become even more puzzling in light of the basic economic principle that society should prefer policies interfering with “prices” (such as the tax rate) rather than “quantities” (such as affirmative action or quotas) in the market.\footnote{In international trade, for instance, a sort of “folk theorem” states that tariffs are weakly superior to import quotas as a trade policy. Taxing polluting activities is generally considered superior to controlling them with quantitative restrictions.}

The optimality of GBT taxation hinges on different elasticities of the labor supply between men and women. If the labor supply elasticity is taken as a primitive, exogenous parameter that differentiates genders, then the argument is quite straightforward. GBT can provide substantial welfare and GDP gains because it satisfies the Ramsey criterion of optimal taxation.

However, one may object that differences in labor market supply functions of men and women, including their elasticities do not only depend on innate characteristics or preferences but emerge endogenously from the internal organization of the family. In fact, as documented for instance by Goldin (2006), Blau and Kahn (2007) and Alabanesi and Olivetti (2007), both the
women participation rate and the elasticity of labor supply, may evolve over time as a result of technologically induced or culturally induced changes in the organization of the family.\textsuperscript{3} Therefore, we also explore a situation in which men and women are identical in terms of innate abilities, preferences and predispositions, but men have more explicit bargaining power at home (possibly for historical reasons). In this case, a gendered allocation of work at home and gender differences in market participation and labor elasticities derive exclusively from the intra-family bargaining. If men have a stronger bargaining power, they assume fewer unpleasant, tiring home duties. Hence, they participate more in the market, exercise more effort, and earn more than their female spouses. The possibility to avoid home duties allows men to engage in careers that offer “upside potential” in terms of wages and promotions. For women, it is the opposite: they basically work only for their wage. As a result, men are less sensitive to changes in the wage since what matters for them, relative to women, is also the intrinsic expected pleasure they derive from careers and market activity. We note that the implied positive correlation between the amount of home duties and the elasticity of labor supply in our model accords well with recent empirical evidence. Aguiar and Hurst (2007) and Blau and Kahn (2007) document a decline in both the ratio of female over male home duties and in the ratio of female over male elasticities of labor supply in the last 50 years.

To the extent that the division of family chores is unbalanced, GBT improves welfare. In addition to satisfying the Ramsey principle of optimal taxation, GBT generates a more equitable allocation of house versus market work. Because of decreasing returns to scale, reallocating “the last hour that the mother spends with the children to the father” is welfare improving for the family as a whole, and under certain conditions it can be welfare im-

\textsuperscript{3}Alesina and Giuliano (2007) study the effect of different cultural traits on family values and ties as a determinant of women participation in the labor force. Ichino and Moretti (2006) show instead how more persistent biological gender differences may affect the absenteeism of men and women and, indirectly, labor market equilibria.
proving for both spouses (as well as for children). Our numerical simulations show that given a difference in the labor supply elasticities calibrated on US estimates (which in the model maps into a corresponding difference in the bargained allocation of home duties), GBT implies rather different tax rates for husbands and wives and can substantially improve welfare, as well as increase GDP and total employment.

An alternative interpretation of our result highlights a difference between short versus long run effects of GBT. The case in which labor supply functions and their different elasticities across gender are exogenous can be interpreted as the short run, namely an horizon in which the family organization and the allocation of home duties can not change. A situation in which the allocation of family duties responds to different tax rates (and this response is internalized by the government in choosing policies) can be interpreted as the long run. In what we call therefore the long run, the family responds to government policies and evolves to a new equilibrium.

It should be noted that the framework that we consider takes a different approach with respect to the literature in modeling household production. The traditional approach builds on the Beckerian theory of the allocation of time (1965), and assumes that household duty is an input to the family production function for the production of a household good. In our model with endogenous gender differences in elasticities we start by a woman and a man who form a family and receive a collection of shocks that must be allocated between the two spouses. With this assumption we intend to capture the fact that there are features of the daily household routine, for example a sick child or a broken dishwasher, that are easy to conceptualize as exogenous but negotiable jobs to be done but not as the output of an intra-household process that transforms time input into a household good. Obviously the two approaches are not mutually exclusive and a more general model of household allocation of time and shocks could capture both aspects of family life.

We further illustrate the link between our model and the literature in
Section 2. Section 3 discusses GBT in the short run, that is when gender differences in labor supply elasticities are held constant and exogenous. In Section 4 we endogenize the allocation of household chores and in Section 5 we show how family bargaining implies an intra-household division of duties, market participation and elasticities. In what we call the long run the government sets taxes anticipating the family’s reaction to fiscal pressure. This is analyzed in Section 6. Section 7 concludes.

2 Related Literature

The present paper lies at the intersection of three strands of research. The first is concerned with the structure of the family. The traditional “unitary” approach, in the spirit of Samuelson (1956) and Becker (1974), treats the household as a single decision making unit. Although this approach is closely linked with the traditional consumer’s theory, it is at odds with the notion of individualism, and, most importantly for our purposes, lacks the proper foundations to conduct intrahousehold welfare analysis. The “collective approach” to family modeling, initiated by Chiappori (1988, 1992) and Apps and Rees (1988), builds instead on the premise that every person has well defined individual preferences and only postulates that collective decisions lie on the Pareto frontier. A more specific approach, taken first by Manser and Brown (1980) and McElroy and Horney (1981), “selects” a specific point on the Pareto frontier by assuming that members of the family Nash-bargain over the allocation of commodities and models the threat points as the utility levels under autarky. Lundberg and Pollak (1993), instead, argue that the threat point can be seen as a (possibly inefficient) non-cooperative equilibrium of the game.

5Two notable empirical failures of the unitary model are the restrictions that arise from the income pooling hypothesis and the symmetry of the Slutsky matrix. See Thomas (1990), Browning, Bourguignon, Chiappori and Lechene (1994), Lundberg, Pollak and Wales (1997), and Browning and Chiappori (1998).
The (long run) model that we consider is in the spirit of the collective approach with Nash-bargained household allocations. The difference with the above models is that the bargaining is not on the allocation of consumption, income and labor supply per se, but on the allocation of home duties. However, our model is set in stages and therefore the bargaining process internalizes the allocation of consumption, labor supply and training. We assume that a certain amount of resources is exogenously shared, and rationalize the sharing parameter as a technological externality that captures the non-excludable and non-rivalrous, at least to some extent, nature of the common consumption of goods within the family.\textsuperscript{6} We study how changes in this parameter affect our results.

The second relevant strand of literature refers to the taxation of couples. The “conventional wisdom” says that under specific assumptions, we should tax at a lower rate goods that are supplied inelastically as suggested by Ramsey (1927). The application of the Ramsey “inverse elasticity” rule in a model of labor supply implies that males should be taxed on a higher tax schedule than females because they have a less elastic labor supply function. This point was made by Rosen (1977) and Boskin and Sheshinski (1983).\textsuperscript{7} Since gender is inelastically supplied, this proposition relates also to the insight that taxes should be conditioned on non-modifiable characteristics as in Akerlof (1978) and Kremer (2003).\textsuperscript{8}

This conventional wisdom regarding lower taxes for women can be chal-

\textsuperscript{6}For example, once the family purchases an electric appliance such as a refrigerator or a dishwasher it is difficult to imagine how a spouse can be excluded from its consumption. Or, the consumption of cable television from one family member does not restrict the consumption of the good by other members of the family.

\textsuperscript{7}The argument was raised using variants of the Diamond and Mirrlees (1971a and 1971b) and Atkinson and Stiglitz (1972) frameworks, also adopted in this paper. Using the Mirrlees (1971) approach, the elasticity of labor supply reappears in the optimal tax schedule, albeit in a less clear way. For an ambitious paper that takes the latter approach see Kleven, Kreiner and Saez (2006), or Kremer (2003) within an application to the problem of age based taxation.

\textsuperscript{8}See Mankiw and Weinzierl (2007) for a recent application of this idea aimed at discussing the validity of the welfarist approach to optimal taxation.
lenged or reinforced in at least three ways. First, it might be the case that the female’s tax rate is a better policy instrument when considering *across* household redistribution. Boskin and Sheshinski (1983) show that this is not the case in their numerical calculations. Recently, Apps and Rees (2007) give intuitive and empirically plausible conditions under which it is optimal to tax males at a higher rate even with heterogeneous households. Second, Piggott and Whalley (1996) raise the issue of intrahousehold distortion of efficiency in models with household production. Since the optimal tax schedule must maintain productive efficiency (Diamond and Mirrlees 1971a), imposing differential tax treatment distorts the intrahousehold allocation of resources and raises a further cost for the society. Although the Piggott and Whalley argument is intuitive, Apps and Rees (1999b) and Gottfried and Richter (1999) show that the cost of distorting the intra-household allocation of resources cannot offset the gains from taxing on an individual basis according to the standard Ramsey principle.

We are interested in exploring the optimality of individual taxes in a model where *within* household redistribution is explicitly taken into account. In that respect our (long run) model is in line and reinforces the conventional wisdom. Earlier models have emphasized that intrahousehold redistributional factors are important. However, these papers are either concerned with the across household heterogeneity (Apps and Rees, 2007), or follow a policy reform approach (Brett, 1998, and Apps and Rees, 1999a), or focus on the positive effects of the taxation of couples (Gugl, 2004). In this paper we explicitly consider the second best problem and search for the globally optimum set of individual tax rates. In doing so, our model focuses on the implications of intrahousehold heterogeneity abstracting from the interhousehold dimension.

The third strand of literature attempts to explain gender differences in labor market outcomes. For example, Albanesi and Olivetti (2006) propose that gender differences can be supported by firms’ expectations that the
economy is on a gendered equilibrium in a model with incentive problems. More traditional theories start by assuming that females have a comparative advantage in home production and males in market production, but Albanesi and Olivetti (2007) show that improved medical capital and the introduction of the infant formula has reduced the importance of this factor. In Becker (1985) gender differences in earnings arise from the fact that females undertake tiring activities that reduce work effort. So, workers with the same level of human capital, earn wages that are inversely related to their housework commitment. The substitutability between home duties and market earnings also arises in our model, although there is also an investment in costly effort effect a la Mincer and Polachek (1974).

Regarding the elasticity of labor supply, Goldin (2006) documents that the fast rise of female’s labor supply elasticity in the 1930-1970 period was the result of a declining income effect and a rising, due to part time employment, substitution effect. During the last thirty years, she argues, females started viewing employment as a long term career rather than as a job, and this caused a decline in the substitution effect and the labor supply elasticity. This interpretation is consistent with how we model, in our long run setting, the elasticity effect of a commitment to stay in the labor market in order to take advantage of the opportunities offered by it. Blau and Khan (2007) also document and quantify the reduction in the labor elasticity of married women in the US, which however remains well above that of men, at a ratio of about 4 to 1.

3 Exogenous Elasticities

A family consists of a male and a female who participate in market and home activities. A costly investment in training makes a person more productive for the market. Husband and wife share a fraction of the income they produce with market work. For the moment we let household activities in the
background and treat them as exogenous.

3.1 Setup of the Model

The index $j = m, f$ identifies the gender. The utility function of gender $j$ is simply given by

$$U_j = C_j - \frac{1}{a_j}L_{aj}^{aj} - \frac{1}{2}\tau_j^2$$  \hfill (1)

where $C_j$ is consumption, $\frac{1}{a_j}L_{aj}^{aj}$ represents the disutility cost of supplying $L_j$ units of labor and $\frac{1}{2}\tau_j^2$ is the cost of training. Each person is endowed with one unit of time for work, so $L_j \leq 1$. The quasilinearity with respect to consumption allows us to obtain closed form solutions at least up to a point. We discuss below the effect of this assumption on our numerical results.

The timing is as follows. First, the government sets labor income taxes. Then, the male and the female take as given the tax rates and decide individually the amount of consumption, labor supply and training to maximize their utilities. A perfectly competitive, constant-returns to scale firm pays workers their marginal productivity and makes zero profits. The price of the consumption good is one and the production function for worker $j$ is

$$Q_j = \tau_j L_j$$  \hfill (2)

Therefore, the wage rate $W_j$ equals $\tau_j$. Spouse $j$ maximizes utility taking as given the labor income tax rate $t_j$ and the other spouse’s decisions

$$\max_{C_j, L_j, \tau_j} U = C_j - \frac{1}{a_j}L_{aj}^{aj} - \frac{1}{2}\tau_j^2$$  \hfill (3)

subject to

$$C_j = s(1 - t_j)W_jL_j + (1 - s)(1 - t_k)W_kL_k$$  \hfill (4)

$$W_j = \tau_j$$  \hfill (5)

where $k$ is the other spouse and $1/2 \leq s \leq 1$ is the sharing parameter. Within a single tax regime, $s$ has the interpretation of an intrahousehold inequality.
parameter. When $s = 1/2$, then the family fully pools its resources and the ratio of consumption levels $C_j/C_k$ equals 1. When $s = 1$, the ratio of consumption levels is pinned down by the ratio of gross incomes, $C_j/C_k = W_jL_j/W_kL_k$, and there is no sharing of resources. Finally, note that in deciding the level of training, workers internalize that a higher level of investment increases their productivity and therefore their wage rate.

The solution to the above maximization problem yields the labor supply and the training decision functions (see the Appendix to Section 3.1 for details)

$$L_j = (s(1 - t_j))^{\sigma_j} = (s(1 - t_j))^{\frac{2\sigma_j}{1 - \sigma_j}}$$

$$\tau_j = (s(1 - t_j))^{\frac{1}{\sigma_j}} = (s(1 - t_j))^{\frac{1 + \sigma_j}{1 - \sigma_j}}$$

where

$$\sigma_j = \frac{\partial L_j}{\partial W_j} \frac{W_j}{L_j} = \frac{1}{a_j - 1}$$

is the own elasticity of labor supply with respect to an exogenous variation in the wage rate. For this Section, cross elasticities are zero because we have assumed quasilinear preferences. In Section 5.4 we also discuss non-zero cross elasticities.

Suppose now that for exogenous reasons we have $a_m > a_f$. For the moment we take this difference in preferences as primitive and do not explain it as it may come from innate gender characteristics or more likely historically induced gender roles which are especially strong in certain cultures (Alesina and Giuliano, 2007). Under $a_m > a_f$ the prediction of the model is that males

- work more in the market: $L_m > L_f$;
- have a lower elasticity of labor supply: $\sigma_m < \sigma_f$;
- invest more in training: $\tau_m > \tau_f$;
• receive a higher wage: $W_m > W_f$.

These predictions are in line with what we observe in real life labor markets. In Figures 1 and 2 we depict the labor market equilibrium. Assuming that $a_m > a_f$, Figure 1 describes a situation in which males supply more labor than females. This happens for two reasons. First, given an exogenous wage rate, male participate more in the market (Becker 1985). Second, they also invest more in training. In turn, investment in training endogenously shifts the labor demand curve up and increases the wage rate $W$. As a result the gender differential in labor market participation and earnings expands. In Figure 2 we describe an exogenous shift in the tax rate $t_j$ for spouse $j$. Taxation distorts both the labor-consumption margin and the decision to invest in training, so that both the labor supply and the labor demand curve shift. The final equilibrium is characterized by lower participation in the labor market and lower pre-tax wage rate.

### 3.2 Gender Based Taxation

The planner sets taxes for the male and the female in order to raise revenues and finance a public good $G$. In doing so, the planner anticipates the private market equilibrium. Let $U_m(t_m, t_f, a_m, s)$ and $U_f(t_m, t_f, a_m, s)$ denote the indirect utility function for the male and the female respectively. In this Section we assume that the planner weights people uniformly, but we revisit this issue in Section 6.1 where it matters more. Then, the planner solves

$$
\max_{t_m, t_f} \Omega = U_m(t_m, t_f; a_m, s) + U_f(t_f, t_m; a_m, s)
$$

---

9We assume that the public good does not provide utility to anyone and the proceedings are not rebated back. This is without loss in generality since the nature of the results (throughout the paper) does not change when we allow for revenues to be distributed in a lump sum way. See Lundberg, Pollak and Wales (1997) for an natural experiment with intrahousehold lump sum transfers.

10Under $\Omega = \frac{1}{1+\epsilon}(U^{1-\epsilon}_m + U^{1-\epsilon}_f)$ with inequality aversion ($\epsilon > 0$), the difference in the resulting tax rates is even more profound. The same holds for the rest of the paper.
subject to the constraint

\[ t_m W_m L_m + t_f W_f L_f \geq G \]  

(9)

**Proposition 1** If \( \sigma_m \leq \sigma_f \), then \( t_m \geq t_f \).

The proof of Proposition 1 and the intermediate derivations are presented in the Appendix to Section 3.2. Here we just mention that (8) and (9) is not a concave program and we have to establish sufficient conditions for the existence of an interior global optimum with \( t_m \geq t_f \).

This proposition is an application of a standard Ramsey (1927) rule. It is welfare enhancing to tax less the “commodity” which is supplied with higher elasticity. The intuition is straightforward. If \( a_m > a_f \), females are more elastic, so distorting their labor and training decisions is more costly for society. In other words, starting from a single tax rate we can always reduce distortions in the labor and the training markets by increasing a little bit \( t_m \) and decreasing \( t_f \) by more.

In Table 1 we present the welfare gains when moving from a single tax to differentiated taxes by gender. Gender Based Taxation (GBT) is not only welfare enhancing but also brings more equality in labor market outcomes. For conservative values of the elasticity ratio such as \( \frac{\sigma_m}{\sigma_f} = \frac{1}{2} \), GBT raises welfare and GDP by more than 1%. For an elasticity ratio of \( \frac{\sigma_m}{\sigma_f} = \frac{1}{3} \) the gain exceeds 4% of GDP.\(^{11}\) Naturally, GBT is more efficient the higher is the level of distortions (i.e. the higher is public expenditure \( G \)) and the lower is the ratio of elasticities \( \frac{\sigma_m}{\sigma_f} \). We defer the discussion of how the resource sharing parameter \( s \) affects the gender taxes for Section 6.2.

The GDP gains are very large, possibly unreasonably so. We also have explored other examples which eliminate the quasi-linearity with respect to consumption. For reasonable parameters and functional forms we find that

\(^{11}\)For evidence on the gender differential on labor supply elasticities see Alesina, Glaeser and Sacerdote (2005), Blau and Kahn (2007) and Blundell and MacCurdy (1999).
with a ratio of elasticities $\sigma_m / \sigma_f = \frac{1}{4}$ (as in Blau and Kahn (2007) for the US), GBT raises GDP by approximately 1.24%.

4 The Organization of the Family

Thus far we have assumed that different labor market behavior of men and women derive from exogenous differences in preferences and attitudes. That is, we have taken the key parameters $a_m$ and $a_f$ as our primitives when conducting our comparative statics. In what follows we propose a possible formalization of the household allocation of home duties from which derives these parameters endogenously.

We assume that there are $2A$ family duties to be undertaken. Each duty is performed by one spouse. When a spouse performs one home duty she/he gets nothing while the other spouse gets a positive shock in the labor market. The argument is similar to that of Becker (1985) who posits that the spouse who does more homework has fewer “energy units” to allocate into the market.

Therefore, there are $2A$ corresponding labor market shocks that hit the family. The shocks are assumed to be i.i.d. and denoted as $x_i$. Each random variable $x_i$ is distributed as a chi-squared with one degree of freedom, i.e. $x_i \sim \chi^2_1$. Let $2a_m$ be the number of $x_i$ shocks that the male absorbs; each shock corresponds to one unit “off-duty” that he gets. $2a_f = 2(A - a_m)$ is the amount of home duties that the male gets, and therefore it is also the number of labor market shocks that the female absorbs. By the properties of the $\chi^2$ distribution we can define an “aggregate shock” for the male as $\omega_m = \sum_{i=1}^{2a_m} x_i$, with support in $[0, \infty)$ and expected value $E(\omega_m) = 2a_m$. Similarly for the female we have that $\omega_f = \sum_{i=2a_m+1}^{2A} x_i$, with support in [12]

For this exercise we use the standard CRRA/power expression for the subutility of consumption which induces both substitution and income effects on labor supply. In this case, the concavity of the subutility function for consumption mutes the welfare gains from the reallocation of resources. For more details see the Appendix to Section 3.2.
and $E(\omega_f) = 2a_f$. Ex post utility for spouse $j = m, f$ is defined over bundles of consumption, labor and training and given by

$$V_j = C_j - \frac{1}{a_j}e^{v(L_j)\omega_j} - \frac{1}{2}\tau_j^2$$

where $C$ is consumption, $L$ is labor supply in the market, and $\tau$ is amount of training. The subutility of labor is given by $v(L_j) = \frac{1}{2} \left( 1 - \frac{1}{L_j} \right) < 0$, with $v' > 0$, $v'' < 0$ and $L_j < 1$. This specific $\chi^2$ shocks - CARA utility environment is adopted to obtain the more familiar CRRA representation of the (ex ante) utility function that we used in Section 3.

To fix ideas about the nature of the shocks, consider the situation where the male and the female decide how to allocate home duties over a period of two weeks. Specifically, for each weekday, one of the two spouses must be in “charge of the kids” (i.e. take them to school, make sure that they have their time after school organized etc.). This hypothetical situation can be mapped in our notation as follows. $2A = 10$ is the total number of days in which one parent has to take the kids to school while the other is exempted from these home duties. $2a_m$ is the number of days that the male is not in charge of the kids and therefore $2a_f$ is the total number of days where the male is in charge of the kids. For each of the $i = 1, ..., 2a_m$ days where the father is not in charge of the kids and works in the market, there is a positive shock $x_i$ that affects his utility of working in the market. To put it differently (and with a slight abuse of language), in the days in which a spouse is not in charge of kids, she/he has more energy and can make “things happen” at work and get a positive utility reward. There are also days in which the spouse is in charge of the children and work provides only the basic wage with no upside options.\(^{14}\)

\(^{13}\)In this sense one cannot “quit a child” and home duty in our model is intrinsically different from having a second job.

\(^{14}\)The abuse of language is that we do not model energy explicitly. Instead, taking less home duties directly implies the possibility of receiving a higher labor market shock.
The expost utility of working in the market for spouse $j$ is given by the term $-\frac{1}{a_j} e^{u(L_j)\omega_j} < 0$. Given a realization of $\omega_j$, a higher amount of labor supply decreases utility. For given amount of labor supply, a favorable realization of $\omega_j$ increases the utility of working in the market (or decreases the disutility of working). Since the shock $\omega_j$ has not been realized when spouses decide how much to consume, supply labor and invest in costly training, we need to work with the ex ante utility function. Using the moment generating function of a chi-squared random variable with $2a_j$ degrees of freedom we obtain

$$U_j = E_\omega V_j = C_j - \frac{1}{a_j} L_j^{a_j} - \frac{1}{2} \tau_j^2$$

(11)

The “$\chi^2$-CARA” expost representation of preferences in (10) allows us to work with the familiar CRRA-power expression for labor supply in (11), which is the utility function used in Section 3 (see equation 1). With this derivation we intend to provide a rationale for the key parameter $a_j$. While in the previous section gender differences were “innately” built in preferences (so that $a_m$ and $a_f$ were “genetically” or “culturally” fixed in a permanent way), in Section 5 we set up a bargaining game that specifies the division of shocks between the two spouses and ultimately determines endogenously their market participation and elasticity.

The expected marginal utility of working is given by

$$U_L = -L^{a-1}$$

with $a > 2$ and $L < 1$

(12)

so that fewer home duties (higher $a$) increase the expected marginal utility of working for spouse $j$. Because the latter expects a higher realization of the labor market shock $\omega$, he or she works more, invests more in human capital and earns a higher wage rate. This means that home duties and participation in the market are substitutes.\(^{16}\)

\(^{15}\)We have that for a random variable $\omega_j \sim \chi^2_{2a_j}$ the moment generating function evaluated at some $q < 1/2$ is given by $M_\omega (q) = E_\omega (e^{q\omega_j}) = \left(1 - \frac{1}{2q} \right)^{a_j}$.

\(^{16}\)This substitutability resembles Becker’s (1985) assumption that the utility cost of
At the same time, taking less home duties implies a higher elasticity of the expected marginal utility of working with respect to labor supply

\[ \varepsilon_{U,L} = \frac{U_{LL} L}{U_L} = (a_j - 1) = \frac{1}{\sigma_j} \]  

(13)

Since for fewer home duties the marginal utility of working is more sensitive to movements in the supply of labor, a given change in the wage rate \( W_j \) meets with a smaller movement in labor supply \( L_j \) in order to restore the first order condition for labor supply. This implies that spouse \( j \) has a less elastic labor supply.

Thus, the gender gap in labor supply elasticities can be traced back to the attitudes of the two spouses towards risk and to the differences in the access to labor market shocks which is determined by the bargained allocation of home duties. For spouse \( j \) and given a specific realization of the labor market shock \( \omega \), we define \( u = -\frac{1}{a} e^{v(L)}\omega \) to be the exposuredisutility from labor supply. We also define the curvature functions \( r_\omega = -\frac{u_\omega}{u} \) and \( r_L = \frac{u_L}{u} \) as measures of the attitude towards risky realizations of \( \omega \) and \( L \) respectively. Then we can show that

\[ \frac{\partial r_\omega}{\partial L} = -\frac{\partial r_L}{\partial \omega} = -v'(L) < 0 \]  

(14)

The first part of the symmetry condition (14) states that a spouse who participates more in the labor market is less risk averse to stochastic realizations of \( \omega \). We can think of this third-order cross partial effect as a diversification motive. High realizations of \( L \) cause spouse \( j \) to be less averse to \( \omega \)-uncertainty since uncertainty “per unit” of labor decreases. The second part of equation (14) states that a spouse getting a good realization of \( \omega \) is more risk averse to stochastic realizations of participating in the market \( L \). \( ^{18} \)

\[^{16}\] effort is increasing in home hours. For a recent discussion of the implications of this assumption see Albanesi and Olivetti (2006).

\[^{17}\] We don’t have a minus sign in the definition of \( r_L \) because labor is a “dis-commodity”, i.e. \( u_L < 0 \). See Appendix to Section 4 for more details.

\[^{18}\] Here the wording “more risk averse” can be rephrased as “less risk lover” because exposet \( r_L \) can be negative or positive depending on the particular realization of \( \omega \). However
The intuition for the gender gap in elasticities is that if $a_m > a_f$, men get fewer home duties and a higher number of shocks to the marginal utility of working. For these reasons they are more willing to commit to a stable labor supply ex ante. In other words, the expected intrinsic utility of working becomes a stronger anchor for the amount of labor that males decide to offer ex ante. When the wage changes, this anchor constraints more firmly their willingness to adjust their labor supply. The reverse if true for women: fewer shocks in the utility of working cause the female to respond more to changes in the wage.\textsuperscript{19}

As we further discuss in the Appendix to Section 4, in this setup the volatility that a spouse faces in the labor market endogenously shifts his or her preferences for labor supply. In turn, the degree of variability that each spouse faces depends on the outcome of the bargaining game that we now analyze.

5 Household Bargaining

5.1 Timing

The timing is as follows. First the government sets the tax rate(s). Then the family members bargain over the allocation of home duties, which results in equilibrium values for $a_j$. Next, labor supply decisions are taken, wages paid, shocks realized and consumption shared. A commitment technology makes it impossible for the government to change the tax rates after family bargaining decisions are made or after labor market shocks are realized.\textsuperscript{17}

\textit{E}$_{\omega}$(r$_L$) is always positive so that every spouse always expects to be an ex post risk averse person in $L$.

\textsuperscript{19}For a foundation of the cross elasticities of labor supply see Section 5.4
5.2 Bargaining over Home Duties

At the second stage of the game spouses decide whether to marry or not marry. If the male and the female decide to marry, then they bargain over the allocation of home duties, \( A = a_m + a_f \). In doing so, they both rationally anticipate the resulting labor market equilibrium. The utility of a spouse \( j \) when married is given by the indirect utility function at stage 3, as described by the maximization of (3) subject to the constraints (4) and (5). We assume that the autarky utility level of each spouse (the threat point), is given by the value function of the following program

\[
\max_{C_j, L_j, \tau_j} T_j = C_j - \frac{1}{\phi} L_j^\phi - \frac{1}{2} \tau_j^2 - z
\]

subject to

\[
C_j = (1 - t_j)W_jL_j \quad \text{and} \quad W_j = \tau_j
\]

This specification of the threat point implies that there is a disutility \( z \) of being alone. On the other hand, a single does not share resources so he or she gets a “full share of a smaller pie”. Importantly, a single has a shock \( \omega_s \sim \chi^2_{2\phi} \) with \( \phi = A \), which means that singles have less home duties than a married person, for instance because they have no children.\(^{20}\) Translated into the words of the example in Section 4, a single person never has to drive the kids to school.

Given this specification of the utilities in marriage and in autarky, for any pair of taxes \((t_m, t_f)\), the allocation of home duties is derived from the maximization of the Nash-product

\[
[U_m(a_m; t_m, t_f, s) - T_m(t_m, \phi, z)]^\gamma [U_f(a_m; t_f, t_m, s) - T_f(t_f, \phi, z)]^{1-\gamma}
\]

where \( \gamma \) is the bargaining power of the male.

\(^{20}\)This assumption can be relaxed. Even when a single has the same amount of home duty with a married person on the equilibrium path, the results do not change. See the Appendix to Section 5.3.
We assume that $\gamma > 1/2$, maybe because in the past physical power mattered and there are persistent cultural forces in the formation of the family.\footnote{The effects and causes of different family structures with specific reference to the role of women and allocation of home duties has been the subject of empirical cross country research by Alesina and Giuliano (2007), and Fernandez (2007). Their results suggest that one should be cautious in applying to different countries and cultures the same set of preferences on the issue of gender roles.} A biased allocation of home duties in favor of the male accords well with our a priori intuition and the existent empirical evidence.\footnote{See Aguiar and Hurst (2007) for recent evidence. Although the difference between male and female house work has decreased during the last 50 years, females still perform nearly twice as much homework as males.} Based on the results of Section 3 with $a_m > a_f$, it also accords well with a situation in which spouses bargain over the allocation of home duties and men have stronger bargaining power, an assumption that also seems consistent with survey evidence. For instance, Friedberg and Webb (2006) use data from the Health and Retirement Study and document that nearly 31% of males believe that “they have the final say in major decisions” while only 12% believe that their spouse is in the same condition. At the same time, approximately 31% of the females admit that their husband has the final say while only 16% believe to have the final say in major decisions.

While our marriage specification is, admittedly, simplified, it well captures the importance of the threat points for GBT. There is a feedback effect from government policy to the intra-household allocation of bargaining power because the outside option of a spouse $j$ depends on the tax rate $t_j$. When for example the tax rate decreases, spouse $j$ acquires more implicit bargaining power through increased training, wage rate and market participation.\footnote{Pollak (2005) argues that the wage rate and implicitly the level of human capital should determine the outside option of a spouse. Our specification addresses, at least partly, this concern because taxes distort the training decision and endogenously shift the labor demand curve.} In the concluding Section 7 we discuss proposed extensions of the marriage market along more realistic lines including a situation where both married couples and singles can exist in equilibrium.
5.3 Properties of the Bargaining Solution

We consider the properties of the solution mapping $a_m(t_m, t_f) : [0, 1]^2 \mapsto (2, A - 2)$. The bargaining solution prescribes how the family allocates home duties for any pair of tax rates, given parameters $\gamma$, $s$, $A$ and $z$. We don’t have closed-form expressions for the solution $a_m(t_m, t_f)$ and its comparative statics, but we can discuss intuitively (and establish numerically) two important properties of the bargaining solution. For more details see the Appendix to Section 5.3.

First, the sharing parameter affects the allocation of home duties. Specifically for given $(t_m, t_f)$, an increase in $s$, i.e. less resource sharing, makes the allocation of shocks more unbalanced, $\frac{\partial a_m}{\partial s} > 0$. We call this the sharing effect and depict it in Figure 3. This Figure plots the male’s indirect utility function $U_m(a_m)$ as a function of home duties. When the male makes take it or leave it offers to the female (equivalent to $\gamma = 1$) and there is no income sharing, he chooses the maximum feasible level of $a_m$, that is he chooses not to take any home duties. As the sharing of resources becomes important ($s$ decreases) the male decides to take some amount of homework, even though he has the maximum level of bargaining power. When income is shared, it is never individually optimal for the male to have the female not working in the market. The same intuition applies for any level of $1/2 < \gamma < 1$. As income pooling becomes more important the intra-household allocation process becomes more balanced. At some level $s_E$, resource sharing is so important that the allocation of shocks is completely balanced, even without government intervention.

The sharing of resources implies that there is an externality in the model. Inspection of the solution (6) suggests that as resource sharing increases, both spouses participate less in market activities because they lose part of their individual claims over the market product. For extreme levels of

\[\text{In that sense this model is a little more individualistic that the collective family model}\]
sharing, $s < s_E$, the male with the bargaining power is better off by staying at home and having the female working and sharing her income with him. This prediction is not realistic and from now on we restrict attention to $s \geq s_E$.

The second property of the bargaining solution can be examined by increasing the tax rate for the male $t_m$ and keeping fixed the female’s tax rate $t_f$ and the level of sharing $s$. Then, from inspection of the Nash product (17) there are three direct effects going in different directions:

- **Redistribution Effect**: $\frac{\partial U_m}{\partial t_m} < 0$. When $t_m$ increases, the male is worse off inside the marriage and demands a lower amount of home duties (higher $a_m$) in order to “stay in the contract”.

- **Threat Effect**: $\frac{\partial T_m}{\partial t_m} < 0$. When $t_m$ increases, the male is worse off outside the marriage and his implicit bargaining power decreases. This means that he is willing to accept a higher amount of home duties (lower $a_m$) in order to “stay in the contract”.

- **Cross Redistribution Effect**: $\frac{\partial U_f}{\partial t_m} < 0$. Because spouses share resources inside the marriage, a higher $t_m$ makes the female worse off inside the marriage. In order to “stay in the contract” she must be compensated with less home duties (lower $a_m$).

We can show (see the Appendix to Section 5.3) that the threat effect always dominates the redistribution effect. That is, a higher tax rate brings a more balanced allocation $\frac{\partial a_m}{\partial t_m} < 0$ because

---

25Even though, for given allocation of home duties, a spouse works and invests less the greater is the sharing of resources, the intra-household allocation process in the bargaining stage of the game is always efficient because the Nash bargaining process is Paretian. Referring to Figure 5, note that the allocation of resources always lies on the Pareto frontier because we cannot make one spouse better off without worsening the position of the other (see also equation (20))

26See the Appendix to Section 5.3 for the robustness of this result after considering the second order effects.
\[
\frac{\partial U_m}{\partial t_m} - \frac{\partial T_m}{\partial t_m} > 0 \quad (18)
\]

which holds if (but not only if) \( s < 1 \) and \( a_m < \phi \). Similar reasoning (but not symmetric because \( \gamma > 1/2 \)) holds for varying the female’s tax rate and \( \frac{\partial a_m}{\partial t_f} > 0 \).

We sum up this discussion in Figure 4 which depicts the solution to the bargaining program as a function of the sharing of resources \( s \) and the ratio of taxes \( t_m/t_f \).

### 5.4 Cross Elasticities

With an endogenous allocation of home duties the cross elasticities of labor supply are not zero as in Section 3. When \( t_j \) changes for spouse \( j \), the allocation of home duties changes endogenously and both spouses adjust their labor supplies. We can write for spouse \( k \)

\[
e_{L_k, t_j} = \frac{\partial L_k}{\partial t_j} \frac{t_j}{L_k} = \left( \frac{\partial L_k}{\partial t_j} + \frac{\partial L_k}{\partial a_k} \frac{\partial a_k}{\partial t_j} \right) \frac{t_j}{L_k} \quad (19)
\]

The term \( \frac{\partial L_k}{\partial t_j}(a_k) \) in (19) is the response of \( k \)’s labor supply to \( j \)’s tax rate for a given allocation of home duties. This is zero as in Section 3 because preferences are quasi-linear and the budget constraint is separable in spouses’ net incomes. The term \( \frac{\partial L_k}{\partial a_k} \frac{\partial a_k}{\partial t_j} \) appears because the allocation of home duties is endogenous and responds to variations in the tax rate. For instance, a higher tax rate for the male \( t_m \), increases the relative bargaining power of the female. As a result, the female takes less home duties (\( a_f \) increases), and the cross elasticity of labor supply with respect to her spouse’s tax rate is positive.\(^\text{27}\)

\(^\text{27}\)This is in line with the empirical evidence, see for example Aaberge and Colombino (2006) for negative cross wage elasticities in Norway and Blau and Kahn (2007) for the US.
6 Gender Based Taxation

Now we examine how the government sets taxes in the “long-run”, that is when the allocation of home duties and therefore the elasticities are endogenously derived.

6.1 Government Objectives

As we discussed in Section 3, GBT is optimal if men are less elastic than females. Based on our derivation in Section 4 this happens when males assume fewer home duties than women, in a model where women have no comparative advantages in home duties.\(^{28}\) It is clear that since the male and the female are identical in everything else but the bargaining power, if \(\gamma = 1/2\) then the market and non-market behavior between spouses is identical and there is no need for GBT. However as discussed before, there is ample evidence for gender differences in labor market participation rates and elasticities and a biased allocation of home duties and decision making power within the family. Given this asymmetry, the crucial question is how the society (i.e. the social planner in our context) should evaluate the utility of husbands and wives. A natural premise is that the social planner evaluates people equally, that is we adopt the utilitarian welfare function, \(\Omega = U_m + U_f\).

If (and only if) there is a “social dissonance” (Apps and Rees 1988) between the preferences of society (as for example implied by the utilitarian function \(\Omega\)) and the equilibrium result of an intrafamily game in which one party has a disproportionate share of power, there is a justifiable reason for the government to intervene in ways which, in addition to financing the public good, affect the intra-family bargaining process.

---

\(^{28}\)Extensions that allow for this possibility are left for future research. Note that Ichino and Moretti (2006) find that biological differences explain a large part of the gender differential in absenteeism which translates in a 12% fraction of the earning gap. Albanesi and Olivetti (2007) point out that technological improvements have certainly reduced women’s comparative advantage in household production and duties.
Note that in this model if the government could choose directly the allocations of home duties and then set taxes to raise a pre-specified amount of revenues, then the ungendered equilibrium \((a_m = a_f, t_m = t_f)\) would be the first best and there would be no need for GBT. In Figure 5, we depict this Edgeworth’s (1897) “egalitarian” solution: remember that we have no comparative advantages of any gender in market or non market activities and we have decreasing marginal utilities. So, starting from a gendered equilibrium \((a_m > a_f)\), we can allocate one more unit of home duty to the male from the female and increase social welfare because there are “decreasing returns to specialization”. In other words, the first hour that the father spends with his children is more productive than the female’s last hour. This is true because starting from \(a_m > a_f\) we have
\[
\frac{\partial \Omega}{\partial a_m} = \frac{\partial U_m}{\partial a_m} + \frac{\partial U_f}{\partial a_m} < 0
\] (20)

The government, however, cannot dictatorially impose a balanced intra-household allocation of shocks; instead it must respect the private sector’s equilibrium. The main message of our analysis however is that, the government can alter the intrahousehold allocation process and achieve a more ungendered equilibrium through gender specific taxes.

\(29\) Even though \(2a_m\) and \(2a_f\) can take only integer values, for expository reasons we discretize the total number of shocks \(A\) into a more continuous grid and treat them as continuous variables when conducting comparative statics. Alternatively, we could increase \(A\) to create meaningful variations in \(a_m\) and \(a_f\), but at the expense of calibrating the elasticities and burdening the notation.

\(30\) Concavity of the indirect utility functions with respect to \(a_m\) is not a global property, but it always holds for the Pareto efficient allocations that we examine. See the Appendix to Section 6.1 for details.

\(31\) Affecting \(a_m\) and \(a_f\), at least to some extent, could be the role for parental leave policies, which, however, can hardly be enforced in reality. See, for example, Friebel, Eckberg and Erickson (2005).
6.2 Gender Based Taxation and Family Organization

The planning program can be written as

\[ \max_{t_m, t_f} \Omega = U_m(t_m, t_f; a_m, a_f, s) + U_f(t_f, t_m; a_f, a_m, s) \]  \hspace{1cm} (21) 

subject to the constraint

\[ t_m W_m L_m + t_f W_f L_f \geq G \]  \hspace{1cm} (22) 

The difference with respect to Section 3.2 is that now the allocation of home duties is endogenous and the government anticipates that by setting taxes it affects the private sector’s equilibrium. That is

\[ a_m = a_m(t_m, t_f; \gamma, s, z) \]

\[ a_f = a_f(t_f, t_m; \gamma, s, z) \]

\[ W_j = W_j(t_j, a_j(t_j, t_f)) \]

\[ L_j = L_j(t_j, a_j(t_j, t_f)) \]

for \( j = m, f \).

We first examine government’s incentives. Starting from a single tax rate, the government can induce a more balanced allocation by differentiating taxes and setting \( t_m > t_f \). As long as labor supply elasticities remain different \((\sigma_f > \sigma_m)\), GBT also reduces fiscal distortions as in Section 3.2.

There is an implicit cost, however, of taxing the male on a higher schedule. By taxing the male at a higher rate not only we distort his labor supply and training decisions (as in Section 3.2) but also we force him endogenously to take more home duties (lower \( a_m \)) which further reduces the government’s ability to extract revenues from the primary earner. This “Laffer curve” effect appears in the first order conditions and increases the ratio of the female over the male marginal revenue (see Appendix to Section 6.2 for further elaboration). It can be inspected by looking at the bliss point of spouse \( j \).
under exogenous and endogenous bargaining. For the former case, the peak of the Laffer curve is given at the point where the elasticity of earnings with respect to the tax rate equals -1

\[
t^b_j = \frac{E_j}{\frac{dE_j}{dt_j}} = \frac{a_j - 2}{2a_j} = \frac{1 - \sigma_j}{2(1 + \sigma_j)}
\]  

(23)

where \(E_j = W_jL_j\) are pre-tax earnings. Notice that if a higher \(t_j\) reduces \(a_j\), then the peak of the Laffer curve shifts to the left. Then, for the endogenous bargaining case we have that

\[
\hat{t}^b_j = \frac{E_j}{\frac{\partial E_j}{\partial t_j} - \frac{\partial E_j}{\partial a_j} \frac{\partial a_j}{\partial t_j}}
\]  

(24)

with \(\hat{t}^b_j < t^b_j\) as long as \(\frac{\partial E_j}{\partial a_j} > 0\) and \(\frac{\partial a_j}{\partial t_j} < 0\) as it is the case for the male.

In Figure 6 we depict the solution for the ratio of optimal gender based taxes \(\frac{t^m}{t^f}\) as a function of the sharing parameter \(s\). There are three areas of interest. In Area I the externality is so high \((s < s_E)\) that the male decides to stay at home. The female works more, earns more, is less elastic and the male enjoys resources mainly from his spouse’s income. As mentioned before, this case does not accord with real life labor markets and we can safely dismiss it.\(^{32}\) In Area II, the male has the bargaining power and without extreme sharing of resources he prefers to assume fewer home duties. As a result he works, invests and earns more than the female. The analysis of Section 4 applies, so the male is also less elastic. In Figure 6 we depict the ratio of labor supply elasticities (that move in the opposite direction of the ratio of home duties) under a single and gender based taxes together with the ratio \(\frac{t^m}{t^f}\). Gender based taxes induce a more balanced allocation of home duties and bring closer to 1 the ratio of elasticities because they increase the implicit bargaining power of the female. Moreover as long as \(\sigma_f > \sigma_m\), the conventional Ramsey principle applies and GBT reduces fiscal

\(^{32}\)See also the references for the empirical failure of the income pooling hypothesis given in Section 2.
distortions. Note that with endogenous bargaining and starting from $\gamma > 1/2$, it is relatively more costly for the society to tax the female than it is in the exogenous bargaining case. The reason is that every extra unit of tax revenues that the government raises from the female further deteriorates her implicit bargaining power and results in a more gendered allocation (see Appendix to Section 6.2 for this argument).

In Area III, $t_m > t_f$ is still optimal. In this region, with less resource sharing and given the intuition of the sharing effect in Section 5.3, the ratio of home duties and the ratio of elasticities diverge even more. However, the ratio of tax rates starts to decline. The intuition for this result is given in Figure 7. This Figure depicts the ratio of wage rates (or training levels) $\frac{W_m}{W_f}$ as a function of the sharing parameter $s$ under a single and gender based taxes. Note that the ratio of optimal taxes $\frac{t_m}{t_f}$ in Figure 6 traces the interhousehold inequality $I = \frac{W_m}{W_f}$ that prevails under a single tax rate in Figure 7. The reason is that GBT reacts to the male over female wage ratio under a single tax rate, which is a measure for the misallocation of home duties without government intervention. At the same time, GBT targets the wage ratio under differentiated taxes, because this is correlated with spouses’ relative decision making power. Since GBT reallocates efficiently the bargaining power between spouses, the ratio of wage rates under GBT, and therefore the relative decision power, shifts down relative to the single tax rate, as shown in Figure 7. At some level of sharing $s_M$, however, the household by its own reduces inequality in earnings and since it is costly for the government to further increase the elasticity of the primary earner, there is no reason why the ratio of taxes should continue to diverge for $s > s_M = .92$.

The reason why resource sharing and inequality under a single tax rate exhibit an inverted U-shaped relationship is the following. Under a high

\footnote{Under a single tax rate the ratio of wages $\frac{W_m}{W_f}$ is higher the more gendered is the allocation of home duties. See equation (31) in the Appendix to Section 6.2.}

\footnote{In particular, we can write autarky utilities as $T_j = \frac{2}{\phi} W_j^2(t_j) - z$.}
level of resource sharing, both the male and the female participate less in the market, and the inequality ratio is low. As resource sharing declines ($s$ increases) both partners participate more, but the male at an increasing rate and therefore inequality starts to rise. Under extremely low levels of income pooling the female starts to participate at an increasing rate, so inequality begins to fall. Even for no resource sharing, i.e. $s = 1$, we always have $W_m > W_f$, so there is always incentive for the government to set $t_m > t_f$. See Appendix to Section 6.2 for more details on this argument.

In Figure 8 we depict the gains in welfare, GDP and employment when moving from a single to gender based taxes as a function of $s$. The gains are maximized when pre-gender based inequality is maximized and start to fall when gender based taxation becomes less necessary as in Area III.

Finally, in Figure 9 we depict the possibility that both spouses gain under Gender Based Taxation.\footnote{While the previous qualitative results are robust to changes in the calibration, Figure 9 is just illustrative. Gender based taxation theoretically may make both spouses better off, but this need not always be the case. For easiness of exposition in Figure 9 we have assumed take it or leave it offers.} If resource sharing is important, both spouses gain when moving from a single to gender based taxes because the female starts to work, train and earn more, a decision which is not internalized by the family when spouses individually decide how much to participate in the market.

This last point is important for the political sustainability of GBT. A crucial issue in any policy reform is the design of a “compensation scheme” where the winners can compensate the losers. For the case of GBT, the compensation from females to males is more natural and easy to imagine than in other policies (e.g. opening up to free trade). If people do not live in families, then GBT makes males worse off and females better off. However, when people share resources within the family, it is possible that GBT makes both spouses better off.\footnote{However, this says nothing about singles, which we believe is a fruitful topic for further research.}
7 Conclusions

In this paper we argue that Gender Based Taxation should be taken seriously into consideration as a potential tax policy. If the bargaining power within the family favors the male, GBT with lower tax rates for females is superior to an ungendered tax rate. In what one could label the “short run”, namely before the family organization adjusts to the new tax regime, GBT reduces tax distortion because of the Ramsey principle according to which one should tax less commodities with higher supply elasticities. When the spouses react to GBT by reoptimizing their bargaining over household duties, GBT leads to a more equitable distribution of household chores and market activities. To the extent that this reallocation does not produce complete equity between male and female and therefore the supply elasticities remain different, GBT is optimal. The reallocation towards more equality of household duties is an additional welfare improving effect if society evaluates the welfare of males and females equally. In the “long run”, the welfare gains of GBT derive both from the Ramsey principle and from a more “efficient” organization of the family that takes into account the decreasing marginal benefits in home versus market activities. In our model GBT is optimal for the couple with both members weighted equally and, for some parameter values, for both members of the couple individually.

Rather than reviewing in more details our results it is worth discussing several important avenues for future research. First we have not considered the possibility of a comparative advantage of females in home production. Although recent empirical evidence (Albanesi and Olivetti, 2007) suggests that gender-specific technological progress makes this assumption less relevant in modern times, it is still “a possibility on the table”. Our intuition for a world in which females have a comparative advantage in home production is that there are two forces going in opposite directions. On the one side, the government does not want to impose lower taxes on women and
encourage female market participation because this would oppose possible increasing returns that the household enjoys when spouses specialize in market and non-market activities. On the other side, imposing higher taxes on females, as discussed in Section 5.3, results in a further deterioration of their implicit bargaining power and opens up the gap in the labor supply elasticities. When elasticities diverge, we expect the Ramsey effect to become stronger and counterbalance the effect of comparative advantages. Which of these two effects prevails is an open issue that requires more theoretical and empirical work.

Second, our model does not allow for a realistic marriage market since it considers a society in which marriage is optimal for everybody along the equilibrium path. A proper discussion of the marriage market would require the introduction of some heterogeneity within the pool of men and women and the consideration of a matching model. In the present model the word “training” can be interchanged with “effort”. The training decision is taken when the couple is already formed. Therefore, we cannot analyze a situation in which a man or a woman, when unmarried, invest in training as a commitment to gain bargaining power. This interesting extension could be discussed in an even more general model in which the marriage market is also endogenized. A key question that this analysis could help answering is whether or not GBT should refer to only married couples or to males and females regardless of their marriage status. An answer to this question would depend undoubtedly on the redistributive properties across families, that the latter solution would imply.

Third, our model does not distinguish between the intensive and extensive margins of labor supply decisions. There is instead an important discontinuity between starting to work from inactivity and increasing working time if someone is already active in the market.

Finally, we believe that a comparison of Gender Based Taxation with other gender and family policies, like quotas, affirmative action, forced parental
leave and public supply of services to the families, is necessary within a uni-
ified theoretical framework in order to draw policy conclusions. We see no
reason why GBT should not be an excellent “horse” in a race with all these
alternative policies. In fact our basic economic intuition regarding the su-
periority of price incentives versus quantity restrictions or regulations would
make GBT a favorite in the race, but we still have to run it.
References


Appendix

Appendix to Section 3.1

Equation (6) is derived by substituting constraints (4) and (5) into the objective function (3), taking the first order conditions with respect to $L$ and $\tau$ and solving the resulting system of equations. The second order sufficient conditions for this maximization problem hold, as $U_{LL} < 0$, $U_{\tau\tau} < 0$, $U_{LL}U_{\tau\tau} - U_{L\tau}^2 > 0$ ($U$ is globally strictly concave in $(L, \tau)$). Equation (7) gives the elasticity of labor supply with respect to an exogenous variation in the wage rate. This is derived from the first order condition with respect to labor supply (for given amount of training), $s(1-t_j)W_j - L_j^s \tau^{-1} = 0$. The comparative statics on wages, labor supply, elasticities and training, starting from $a_m > a_f$ follow directly from inspection of (6) and (7).

Appendix to Section 3.2

We first substitute the solution (6) and the constraints (4) and (5) into the objective function (3) and derive the indirect utility function. Denote $R_m = t_m L_m(t_m; a, s)W_m(t_m; a, s)$ and $R_f = t_f L_f(t_f; a, s)W_f(t_f; a, s)$ the revenues collected from the male and the female respectively evaluated at the solution (6). Then we can write the planning program as

$$\max_{t_m, t_f} \Omega = U_m(t_m, t_f; a, s) + U_f(t_f, t_m; a, s)$$

subject to the constraint

$$R = R_m + R_f \geq G$$

A standard complication in public economics (Diamond and Mirrlees 1971b, Myles 1995, pp 113-114) arises from the fact that the above maximization problem is not sufficiently “smooth”. The problem arises in the dual approach because the consumer’s indirect utility function is quasiconvex in prices (and income). The program can be turned into a concave problem for a social welfare function of the form $\Omega(U_m, U_f)$, with $\Omega$ being sufficiently concave (high inequality aversion), but in general the transformation of a concave with a convex function is not guaranteed to be concave. In our case, with quasilinear preferences and the utilitarian welfare (i.e. $\Omega U_m = \Omega U_f = 1$), welfare is strictly convex in the tax rates. This means that we cannot simply invoke standard sufficient conditions from the theory of concave programming.

To establish the sufficiency of the first order conditions for the above problem we follow fairly standard steps.\(^{37}\) First, from the definition of the indirect utilities

\(^{37}\)We also can show that this is true for any welfare function that is more concave than the utilitarian case (which is the least concave welfare function).
$U_m$ and $U_f$ it is straightforward to show that the welfare function $\Omega = U_m + U_f$ is strictly decreasing in $t_m$, strictly decreasing in $t_f$ and strictly convex in $(t_m, t_f)$. So, in the $(t_m, t_f)$ space, the gradient vector $\nabla \Omega$ points towards the origin $(0,0)$ and the lower contour set of the social indifference curve $\Omega(t_m, t_f) = \bar{\Omega}$ is strictly convex.

Second, consider the revenue function for spouse $j$, $R_j$. We have that $\frac{\partial R_j}{\partial t_j} = sL^2_j[1 - t_j - t_j(a_j + 2a_j^2)]$. The peak of the Laffer curve for spouse $j$ comes at the tax rate where the elasticity of earnings with respect to the tax rate is minus unity, so that $t^b_j = \frac{a_j^2 - 2}{2a_j}$. Now we also have that $\frac{\partial^2 R_j}{\partial t_j^2} = sL^2_j[-1 - \frac{a_j + 2}{a_j - 2}] + 2sL_j \frac{\partial L_j}{\partial t_j} [1 - t_j - t_j(a_j + 2a_j^2)]$.

The first term is negative while the second term is negative if $\frac{\partial R_j}{\partial t_j} > 0$. So, the revenue function for spouse $j$ is concave if (but not only if) we are at the upwards sloping part of the Laffer curve. Given the properties of $R_m$ and $R_f$, total revenues $R = R_m + R_f$ are strictly increasing in each of $t_m$ and $t_f$ and strictly concave in $(t_m, t_f)$ if (but not only if) $(t_m, t_f) < (t^b_m, t^b_f)$. This means that in the $(t_m, t_f)$ space the gradient vector of the revenue function $\nabla R$ points towards the bliss point and the upper contour set of the revenue isolevel $R = G$ is strictly convex in that region. Notice if $a_m > a_f$, the bliss point lies above the 45 degree line which signals that $t_m > t_f$ holds in the solution. So, if the government wants to extract the maximum revenue the solution is $t_m = t^b_m > t_f = t^b_f$. Also we define $G_{max}$ to be the maximum sustainable level of public expenditure with $G_{max} = R(t^b_m, t^b_f)$.

Next, it is easy to show that $t_m > t^b_m$ or $t_f > t^b_f$ can never solve the program. If this was not the case, then we could increase both welfare and revenues which contradicts optimality. Therefore, without loss in generality we now restrict attention to the set $D = \{ (t_m, t_f) : t_m \in [0, t^b_m], t_f \in [0, t^b_f], G_{max} \geq R \geq G \}$. Since $D$ is a compact set and $\Omega$ is a continuous function, by Weierstrass Theorem, a global maximum exists in $D$. Finally, it must be the case the constraint always binds at the optimum. If this was not the case, then we could increase welfare by decreasing some tax rate, while still satisfying the constraint.

Fix an arbitrary level of public expenditure. Since we know that $t_m > t_f$ if $G = G_{max}$ we now restrict to $G < G_{max}$. The next step is to establish that for $a_m > a_f$, i.e. $\sigma_f > \sigma_m$, the solution $t_f > t_m$ is never optimal. To show that this cannot be an optimum, it suffices to show that the slope of the welfare function in the $(t_m, t_f)$ space is always greater in absolute value than the slope of the revenue function at every point along the $R = G$ level where $t_f > t_m$. The reason is that then we can decrease $t_f$, increase $t_m$, increase welfare and still satisfy the budget constraint. $t_f > t_m$ is never optimal because the relative marginal cost of taxing

---

38Because labor supply and training are always in the interior all inequalities are strict.
Appendix to Section 4

We have defined \( U \), the slope of the welfare indifference curve by

\[-E \] larger than one larger than \( \Omega \) curve is steeper than the revenue level at any point where \( t_f > t_m \) holds.

Similarly, we can establish that the only point where we cannot increase welfare without violating the constraint is the tangency point (notice however that we had to go through this argument first). In that point the welfare indifference curve is less convex than the budget constraint and the optimal taxes satisfy the condition

\[
\frac{s + (1 - s) (\frac{2\sigma_j}{a_j - 2})}{s + (1 - s) (\frac{2\sigma_m}{a_m - 2})} = \frac{1 - t_f (\frac{a_j + 2}{a_j - 2})}{1 - \frac{t_m (\frac{a_m + 2}{a_m - 2})}{s} + (1 - s) (\frac{2\sigma_m}{a_m - 2})}
\]  

Equation (27) establishes that if \( \sigma_f > \sigma_m \) then \( t_m > t_f \). The tangency condition is unique because the utility function is strictly concave. This ensures that the objective function (25) is strictly convex and the constraint (26) is strictly concave in the tax rates.

For the version of the model without quasilinear preferences, the utility function for spouse \( j \) is given by \( U_j = X_j - \frac{1}{a_j} L^{a_j} - \frac{t_j}{2} \), where \( X_j \) is a ”composite commodity” given by \( X_j = s \frac{C_j}{1 - \theta} + (1 - s) \frac{C_j}{1 - \theta} \) and the budget constraint is simply \( C_j = (1 - t_j) W_j L_j \). In Table 1, we set \( \frac{\sigma}{GDP} = 20\% \) and \( \theta = 0.5 \).

Appendix to Section 4

We have defined \( U = E_\omega V = C - \frac{1}{a} L^a - \frac{1}{2} \tau^2 \) as the expected utility function which is derived under the properties of the chi-squared distribution. Then, equations (12)-(13) are obvious. \( u = -\frac{1}{a} v(L) \omega \) is the expost disutility from labor supply (in \( V = C + u - \frac{1}{2} \tau^2 \)), where \( v(L) = \frac{1}{2} (1 - \frac{t}{L}) \), with \( v' > 0, v'' < 0, v''' > 0 \). We also define the curvature functions \( r_\omega = -\frac{u_{\omega \omega}}{u_\omega} = -v(L) \) and \( r_L = \frac{u_{\omega L}}{u_L} = \frac{v'(L)}{v(L)} + v'(L) \omega \).

While we have that \( r_\omega > 0 \), so spouses are always risk averse in \( \omega \)-variations, \( r_L > 0 \) only for \( \omega > 4L \). However, every spouse expects to be expost averse to \( L \)-variations because \( E_\omega r_L = \frac{v''(L)}{v(L)} + 2av'(L) > 0 \). \( r_\omega \) is constant in \( \omega \) (hence the terminology ”CRA”) but changes with \( L \). \( r_L \) is not constant but depends on \( L \) and \( \omega \). For the third order effect \( \frac{\partial^3 r_L}{\partial L^3} \), a sufficient, but not necessary, condition for this to be

38
negative ("risk prudence") is that \( a_j > 3 \).

What matters for our results is the, in expectation, variation of \( r_L \) with \( \omega \). More specifically, \( E_\omega r_L \) is positively correlated with the \( \text{Var}(\omega) \) because

\[
\frac{\partial E_\omega(r_L)}{\partial \text{Var}(\omega)} = \frac{1}{4L^2} > 0
\]

(28)

For this we have also used the fact that the variance of a chi-squared random variable with 2\( a \) degrees of freedom equals 4\( a \). (28) says that the spouse who gets a higher number of shocks, i.e. faces more variability in the labor market, expects to be exposed more concave in \( L \). This flattens the (ex ante) utility contours (in the \((C, L)\) space) and lowers the elasticity of labor supply.

**Appendix to Section 5.3**

The derivative of the indirect utility function for the male with respect to \( a \) is given by

\[
\frac{\partial U_m}{\partial a} = \frac{1}{2} L^a |\frac{1}{a} - \ln L| + (1 - s)(1 - t_f) \frac{\partial E_f}{\partial a},
\]

where \( E_f = W_f L_f \) are female’s earnings. In the absence of sharing we have \( \frac{\partial U_m}{\partial a} > 0 \). In the presence of sharing, the second term tends to lower \( \frac{\partial U_m}{\partial a} \) because the male loses consumption by forcing the female to stay at home. For extreme levels of sharing, \( \frac{\partial U_m}{\partial a} < 0 \). This is Area I in Figure 6. Similarly for the female. From now on we restrict the discussion in Areas II and III, with \( \frac{\partial U_m}{\partial a} > 0 \) and \( \frac{\partial U_f}{\partial a} < 0 \).

Write the first order condition for the maximization of (17) as \( F(a, t_m, t_f, s, \gamma) = \gamma \frac{\partial U_a}{\partial a} + (1 - \gamma) \frac{\partial U_f}{\partial a} = 0 \). Differentiating this identity with respect to \( a \) we get that

\[
-\frac{\partial F}{\partial a} = \gamma \left[ \frac{\partial U_m}{U_m - T_m} \right] \left[ \frac{\partial U_m}{U_m - T_m} \right]^2 - \gamma \frac{\partial^2 U_m}{U_m - T_m} \left[ \frac{\partial U_m}{U_m - T_m} \right]^2 + (1 - \gamma) \left[ \frac{\partial U_f}{U_f - T_f} \right] \left[ \frac{\partial U_f}{U_f - T_f} \right]^2 - (1 - \gamma) \frac{\partial^2 U_f}{U_f - T_f} \left[ \frac{\partial U_f}{U_f - T_f} \right] = 0.
\]

Since strong individual rationality holds (\( U_m > T_m \) and \( U_f > T_f \)), a sufficient but not necessary condition for \( -\frac{\partial F}{\partial a} > 0 \) is that \( U_m \) and \( U_f \) are concave in \( a \), which is true in the Pareto area (see Appendix to Section 6.1) - the second order condition holds.

Differentiate the first order condition with respect to \( \gamma \) and get that

\[
\frac{\partial F}{\partial \gamma} = \frac{\partial U_m}{U_m - T_m} - \frac{\partial U_f}{U_f - T_f} > 0.
\]

Therefore, using the second order condition \( \frac{\partial^2 F}{\partial \gamma^2} = \frac{\partial E_f}{\partial a} > 0 \), and naturally the male gets less home duties the larger is his bargaining power.

For the sharing effect the second-order effects are too complicated to yield a meaningful comparative static. However, in all our results the first order effect, i.e. that as sharing increases, the male wants to induce work effort from the female and takes more home duties, dominates (see Figures 3 and 4 for an example) and for reasonable perturbation of parameters we have \( \frac{\partial a}{\partial r_f} > 0 \).

What matters for the argument that gender based taxes change the implicit bargaining power is that \( \frac{\partial a}{\partial r_f} < 0 \). However, the intuition may well be inspected
by changing one tax rate at the time.

The redistribution, threat and cross redistribution effects follow from simple inspection of the utilities under marriage and under autarky. That the threat effect dominates the redistribution effect can be established by differentiating $U_m$ and $T_m$ to obtain

$$\frac{\partial U_m}{\partial t_m} - \frac{\partial T_m}{\partial t_m} = -(s(1-t_m)) \frac{\partial E_m}{\partial a_m} - (-1 - (1-t_m)) \frac{\partial E_f}{\partial a_f} > 0$$

which holds if (but not only if) $s < 1$ and $a_m < \phi$. A weaker sufficient condition is that a single person takes less home duties than a married person, which we believe is a reasonable condition. This condition becomes sufficient and necessary for no resource sharing, $s = 1$.

That $\frac{\partial U_m}{\partial t_m} - \frac{\partial T_m}{\partial t_m} > 0$ is "almost" sufficient for $\frac{\partial E_m}{\partial t_m} < 0$ can be established as follows. We want to show that $\frac{\partial F}{\partial t_m} < 0$. For this write $\frac{\partial F}{\partial t_m} = \gamma - \frac{\partial U_m}{\partial t_m} \cdot \frac{\partial U_m}{\partial t_m} - \frac{\partial T_m}{\partial t_m} + \gamma \frac{\partial U_m}{U_m - T_m} + (1 - \gamma) \frac{\partial U_m}{U_m} \cdot \frac{\partial U_m}{\partial t_m} + (1 - \gamma) \frac{\partial^2 U_m}{U_m - T_m}$. We have that $\frac{\partial U_m}{\partial a} > 0$ and $\frac{\partial U_m}{\partial a} < 0$. Now the term $\frac{\partial U_m}{\partial t_m} - \frac{\partial T_m}{\partial t_m}$ is positive because the threat effect dominates the redistribution effect. The term $\frac{\partial U_m}{\partial t_m}$ is the cross redistribution effect and it is negative. The term $\frac{\partial U_m}{\partial a}$ by virtue of the Envelope and Young’s Theorems can be written as $\frac{\partial U_m}{\partial a} = -s \frac{\partial E_m}{\partial a}$ and it is negative as earnings decrease with home duties. Finally, the term $\frac{\partial U_m}{\partial t_m} = -(1 - s) \frac{\partial E_m}{\partial a} - (1 - t_m) \frac{\partial^2 E_m}{\partial a \partial t_m}$ may be either positive or negative, depending on the elasticity of earnings with respect to home duties. If it is negative, which holds for $s$ not very large (say in Area II), then $\frac{\partial F}{\partial t_m} < 0$ as wanted. If it is positive, then $\frac{\partial F}{\partial t_m} < 0$ cannot be established analytically, but in our numerical results the last effect never dominates the three first effects. The reason is that for $s$ very large which is necessary for $\frac{\partial E_m}{\partial a} - (1 - t_m) \frac{\partial^2 E_m}{\partial a \partial t_m} < 0$ to hold, $(1 - s)$ times $\frac{\partial E_m}{\partial a} - (1 - t_m) \frac{\partial^2 E_m}{\partial a \partial t_m}$ becomes negligible. In the absence of sharing, $s = 1$, $\frac{\partial F}{\partial t_m} < 0$ always holds because the first and the second terms are always negative, and therefore $\frac{\partial a}{\partial t_m} < 0$, $\frac{\partial a}{\partial t_f} > 0$ can be examined similarly.

Appendix to Section 6

We denote $a_m = a$ and $a_f = A - a$. We calibrate the total number of home duties to be 20 ($A = 10$). This delivers elasticities of labor supply around .2 for the male and .3 for the female under no resource sharing. The bargaining power $\gamma$ is set at 3/4 because (i) with resource sharing and (ii) spouses being "risk averse in $a^2$", the allocation of resources is quite balanced. For example, with $s = 1$ (i.e. the male willing to take as less home duties as possible) and $z = .2$, the male extracts a little more than 60% of the marriage surplus. For $s < 1$ this is even smaller and changing the bargaining power does not create significant variation in the results. Similarly for $z$ (subject to maintaining strong individual rationality).
In drawing Figures 6-9 we keep constant public expenditure as a percentage of GDP. The reason is that GDP falls quickly with a declining s (both spouses work less), and therefore holding constant the level of public expenditure G results in unmeaningful comparisons. G/GDP is set at 20%.

Appendix to Section 6.1

We don’t have an analytic expression for the solution of the bargaining program. Working numerically and intuitively, the first point is that \( U_m \) is not globally concave in \( a \). Taking the second derivative with respect to \( a \) we have, for example for the male, that
\[
\frac{\partial^2 U_m}{\partial a^2} = aL^{a-1}[\frac{1}{2} - \frac{\ln L}{a^2}] + L^a[\frac{2}{a^2} - \frac{\partial^2 E_j}{\partial a^2} + \frac{\partial^2 E_j}{\partial t^2}] + (1 - s)(1 - t)\frac{\partial^2 E_j}{\partial a^2},
\]
where \( E_j = W_jL_j \) are the female’s earnings. The first term is positive, the second and the third terms negative. However, except for very extreme levels of \( a \) (close to 2 or close to A-2), concavity is ensured. In our numerical results, \( U_m \) is concave in \( a \) everywhere in the Pareto efficient area (i.e. when (20) holds, see Figure 5). Similarly for the concavity of \( U_f \). Since the Nash-bargained allocations are by assumption Pareto efficient, concavity in the area of interest is assured, and the bargaining solution is well defined.

Appendix to Section 6.2

The first order necessary condition for interior local optimum for the program (21)-(22) is given by
\[
\frac{\partial U_f}{\partial t_f} + \frac{\partial U_m}{\partial t_f} + \left( \frac{\partial a}{\partial t_f} \right) \left( \frac{\partial U_m}{\partial a} + \frac{\partial U_f}{\partial a} \right) = E_f + t_f \frac{\partial E_f}{\partial t_f} + t_f \frac{\partial a}{\partial t_f} \frac{\partial E_f}{\partial a}
\]
\[
E_m + t_m \frac{\partial E_m}{\partial t_m} + t_m \frac{\partial a}{\partial t_m} \frac{\partial E_m}{\partial a}
\]
(29)

where \( E_j = W_jL_j \) are gross earnings. This condition says that at the optimum the female over the male ratio of social marginal cost should equal the ratio of marginal revenues that the government can extract from each spouse respectively. Multiplying by \( \frac{1 - t_f}{1 - t_f} \) both sides we can rewrite the first order condition as
\[
\frac{[\frac{1}{1-t_f}(\frac{\partial U_f}{\partial t_f} + \frac{\partial U_m}{\partial t_f})] + [\frac{1}{1-t_f}](\frac{\partial a}{\partial t_f} \frac{\partial U_m}{\partial a} + \frac{\partial U_f}{\partial a})}{[\frac{1}{1-t_m}(\frac{\partial U_m}{\partial t_m} + \frac{\partial U_m}{\partial t_m})] + [\frac{1}{1-t_m}](\frac{\partial a}{\partial t_m} \frac{\partial U_m}{\partial a} + \frac{\partial U_f}{\partial a})} = \frac{[\frac{1}{1-t_f}](E_f + t_f \frac{\partial E_f}{\partial t_f}) + [\frac{t_f}{1-t_f}] \frac{\partial a}{\partial t_f} \frac{\partial E_f}{\partial a}}{[\frac{1}{1-t_m}](E_m + t_m \frac{\partial E_m}{\partial t_m}) + [\frac{t_m}{1-t_m}] \frac{\partial a}{\partial t_m} \frac{\partial E_m}{\partial a}}
\]
(30)

While certainly not sufficient this condition can shed some light in the workings of the solution. In the left hand side, the first terms in the numerator and the denominator are the same as in the case of the exogenous bargaining problem (as
in equation (27)). The second terms in the numerator and the denominator appear because the government desires to affect the allocation of home duties. The term in the brackets \( \frac{\partial U_m}{\partial a} + \frac{\partial U_f}{\partial a} \) is common in the numerator and the denominator. This would have been the first order condition if the government could affect \( a \) directly. Starting from \( a > A - a \) (i.e. the male getting less home duties) this term is negative because of decreasing returns of specialization (at least, in the Pareto area). From the analysis in Section 5.3 and in this Appendix the term \( \frac{\partial a}{\partial t_f} \) in the numerator is positive and the term \( \frac{\partial a}{\partial t_m} \) in the denominator is negative.

Therefore, relative to the case with exogenous bargaining, the ratio of the female’s to the male’s social marginal cost of taxation \( \frac{\partial \Omega}{\partial t_f} \frac{\partial \Omega}{\partial t_m} \) increases. With endogenous bargaining and starting from \( \gamma > 1/2 \) it is relatively more costly to tax the female than it is in the exogenous bargaining case. Every unit of tax revenues that the government raises from the female further deteriorates her implicit bargaining power and results in a more gendered allocation. This intuition is in the heart of the \( t_m > t_f \) result in Section 6.2.

Things however are complicated by the fact that the ratio of marginal revenues also changes relative to the exogenous bargaining case. The difference stems from the last terms in the numerator and the denominator of the right hand side of (30). The term \( \frac{\partial a}{\partial t_j} \frac{\partial E_j}{\partial a} \) measures the shift in the peak of the Laffer curve for spouse \( j \) due to the shift in the intrahousehold allocation of resources. For example, increasing the male’s tax rate results in less bargaining power for the male who has to ”settle in” with a smaller \( a \). Then the male participates less in the labor market and becomes less risk averse, per the intuition of Section 4. This increases his labor supply elasticity, which poses an extra cost for the society since the government wants to tax the male. Relative to the exogenous bargaining case, the last terms in the numerator and the denominator, in general raise the female over the male ratio of marginal revenues. The reason why this appears to be true is that for \( a_m > a_f \) we have that \( \frac{\partial E_j}{\partial a} \) is greater than \( \frac{\partial E_m}{\partial a} \) in absolute value because earnings are concave in \( a \). Also in our simulations \( \frac{\partial a}{\partial t_m} \) seems to be less responsive than \( \frac{\partial a}{\partial t_f} \) due to the bargaining power of the male. If the ratio of the marginal revenues increases, then it is less easy to extract revenues from the male in the endogenous bargaining case. The simultaneous increase of the ratio of marginal costs and the ratio of marginal revenues under endogenous bargaining, prohibits us from comparing the optimal solution \( t_m, t_f \) under the two regimes.

Finally, the relationship between pre-gender based taxation inequality and the sharing parameter \( s \) can be examined by writing the inequality ratio as

\[
I = (s(1-t)) \frac{a_m}{a_m-2} - \frac{A-a_m}{A-a_m-2} \quad (31)
\]
The first point is that since for \( s = 1 \) and \( \gamma > 1/2 \) we always have \( a_m > A - a_m \), we get that \( I(s = 1) > 1 \). Second, for a given level of \( t \) that raises revenues equal to \( G \), let’s call \( K(s) = \frac{a_m(s)}{a_m(s)^2 - 2} - \frac{A - a_m(s)}{A - a_m(s)^2 - 2} \). Since \( a'_m(s) > 0 \), we have that \( K'(s) < 0 \). The two opposite forces of \( s \) on \( I \) can been illustrated as follows. For given \( K(s) < 0 \), a higher \( s \) decreases \( I \) because the female participates more in order to balance the weaker sharing of resource. For given \( s(1 - t) \), a higher \( s \) causes \( K(s) \) to become more negative and this tends to increase inequality \( I \). This is because the male shares less resources with the female and “exerts” his bargaining power by choosing an even more unbalanced home duties ratio. The two forces exactly cancel out at point \( s_M = .92 \) in Figures 6 and 7.
Table 1: Welfare effects of Gender Based Taxation with exogenous bargaining

<table>
<thead>
<tr>
<th>Focus</th>
<th>Tax regime</th>
<th>Parameter values</th>
<th>Endogenous ratios</th>
<th>Gains (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\frac{t_f}{GDP}$</td>
<td>$\sigma_m$</td>
<td>$\sigma_f$</td>
</tr>
<tr>
<td>G</td>
<td>GBT</td>
<td>18%</td>
<td>1.83</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>single</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>GBT</td>
<td>22%</td>
<td>1.83</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>single</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>GBT</td>
<td>20%</td>
<td>1.83</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>single</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>GBT</td>
<td>20%</td>
<td>2.58</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>single</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: In the first three rows elasticities are $\sigma_m = 0.1$ for the male and $\sigma_f = 0.2$ for the female. For the last row we have $\sigma_m = 0.089$ and $\sigma_f = 0.267$ respectively. In all cases we keep the total number of shocks $A$ constant. For the version of the model with CRRA subutility for consumption, for elasticities $\sigma_m = 0.07$ and $\sigma_f = 0.28$ we find the ratio of optimal taxes to be $\frac{t_m}{t_f} = 1.62$, welfare gains of approximately 0.26%, employment gains of around 0.93%, gains in training of 0.42%, and GDP gains of 1.24%.
Figure 1: The Labor Market

- **Labor Wage Rate**
  - **Males**
  - **Females**

![Labor Market Graph]

- **Axes**:
  - **X-axis**: Labor
  - **Y-axis**: Wage Rate

- **Legend**:
  - **Males**
  - **Females**

The graph illustrates the wage rate for males and females in relation to labor.
Figure 2: The Effects of Taxes on the Labor Market

The graph illustrates the impact of taxes on the labor market. On the y-axis, the labor wage rate is shown, ranging from 0 to 1. The x-axis represents labor, ranging from 0 to 1. Two lines are plotted: a solid line labeled 'low tax' and a dashed line labeled 'high tax'. The solid line indicates a lower wage rate at a given level of labor compared to the dashed line, depicting the higher wage rate under high tax conditions. The graph indicates that as labor increases, the wage rate decreases under both tax scenarios, with the high tax scenario showing a steeper decline.
Figure 3: Sharing of Resources and Allocation of Shocks

Utility (male) vs $a (m)$ for different values of $s$: $s=1$, $s=0.9$, $s=0.8$.
Figure 4: The Bargaining Solution

Fraction of Home Duties to Female

\[ \frac{tm}{tf} = 0.85 \]

\[ \frac{tm}{tf} = 1 \]

\[ \frac{tm}{tf} = 1.15 \]
Figure 5: Ungendered Equilibrium is the First Best
Figure 6: $t_m$, $\sigma_m$, $s$; $\gamma = 3/4, z = 0.2, \frac{G}{W} = 20\%$
Figure 7: Wage Ratios - \( s; \gamma = \frac{3}{4}, z = 0.2, \frac{G}{GDP} = 20\% \)
Figure 8: Gains - $s; \gamma = 3/4, z = 0.2, \frac{G}{\text{GDP}} = 20\%$
Figure 9: Both Spouses May Be Better Off with GBT