Understanding the Puzzling Effects of Technology Shocks

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Introduction

- RBC theory: technology expansionary.

- Gali (AER 1999) and Basu et al. (AER 2006): technology contractionary for $I_t$ & $N_t$.

- Two implications: (i) technology shocks not main driving force; (ii) sticky prices.

- "the RBC theory is dead" (Francis and Ramey, JME 2005).
● It is possible that technology shocks not important and prices sticky.

● However, the finding of Gali and Basu et al. does not logically imply these are indeed the case.

● (i) the sign of the initial impulse responses to technology shocks does not imply lack of procyclicality.

● (ii) contractionary effect of technology shocks does not necessarily reject flexible prices – the main ficus of our paper.
In what follows, we first present empirical regularities that appear to be profoundly inconsistent with flexible prices. Then we show that this is not the case.

Stylized Facts

\[
\begin{align*}
1 & \quad 0 & x_t & = a_1 & 0 & x_{t-1} & + a_2 & 0 & x_{t-2} & + \varepsilon_t \\
-c_0 & \quad 1 & y_t & = c_1 & b_1 & y_{t-1} & + c_2 & b_2 & y_{t-2} & + v_t \\
\end{align*}
\]

\[
\begin{align*}
1 & \quad 0 & x_t & = a_1 & d_1 & x_{t-1} & + a_2 & d_2 & x_{t-2} & + e_{xt} \\
-c_0 & \quad 1 & y_t & = c_1 & b_1 & y_{t-1} & + c_2 & b_2 & y_{t-2} & + e_{yt} \\
\end{align*}
\]
Figure 2. Sectorial Response to Agg. Tech. Shock
Figure 3. Response of Real Wage and Real Rate.

\[ \Phi \alpha \frac{Y}{K} = r, \Phi (1 - \alpha) \frac{Y}{N} = w. \]
Figure 4. Distribution of Correlations
Fig 5. Sectorial Response to Sector-Specific
Why tech shock contractionary and asymmetric?

Our approach: Leontief technology at the firm level, with firm entry and exit. Prices fully flexible.

Our model provides micro foundation to aggregate production functions, and is identical to a standard frictionless RBC model in aggregate dynamics if no time-to-build.

However, with time-to-build, our model is able to explain all of the aforementioned empirical facts.
Benchmark Model

Final Good ($y$)

- Identical producers $i \in [0, \Omega_t]$, each producing one unit of final good. (Imagine a production assembly line with fixed production capacity.)

- Entry cost = $\Phi$. Prob of exist = $\theta_t$. Zero profit $\Rightarrow$ total number of producers $\Omega_t$.

- Production function: $y = x$. Normalization: $p_y = 1$. 
• Demand for input:

\[ x = \begin{cases} 
1 & \text{if } p_x \leq 1 \\
0 & \text{if } p_x > 1 
\end{cases} \]

• Profit:

\[ \pi = \begin{cases} 
1 - p_x & \text{if } p_x \leq 1 \\
0 & \text{if } p_x > 1 
\end{cases} \]

• Aggregate supply of output: \( Y = \int_{0}^{\Omega} ydi = \Omega \), aggregate demand for input is \( \int_{i=0}^{\Omega} xdi = \Omega \).
Intermediate Good: (flour)

Final Good: (pizza)

Aggregate Output: (# of pizzas)

\[ Y = y + y + \ldots + y = \Omega y \]

\[ X = AF(K, N) \]

\[ Y(x) \quad Y(x) \quad \ldots \quad Y(x) \]

\[ x = \alpha \text{ units} \quad x = \alpha \text{ units} \quad \ldots \quad x = \alpha \text{ units} \]

Figure 6. Production Structure.
The value of a firm (with time-to-build):

\[ V_t = \beta E_t \Lambda_{t+1} \pi_{t+1} \]

\[ + E_t \sum_{j=1}^{\infty} \beta^{j+1} \left( \prod_{i=1}^{j} (1 - \theta_{t+i}) \right) \Lambda_{t+j+1} \pi_{t+j+1}, \]

\[ \Rightarrow \]

\[ V_t = \beta E_t \Lambda_{t+1} (\pi_{t+1} + (1 - \theta_{t+1}) V_{t+1}). \]

Free entry \( \Rightarrow V_t = \Phi. \)

Evolution of \( \Omega: \)

\[ \Omega_{t+1} = (1 - \theta_t) \Omega_t + s_t, \]

where \( s = \) new entrants.
**Intermediate good**

- Infinitely many identical intermediate good producers, with production function:

\[ X_t = A_t K_t^\alpha N_t^{1-\alpha}. \]

- Profit maximization gives

\[ \alpha p_x \frac{X}{K} = r_t + \delta, \]

\[ (1 - \alpha) p_x \frac{X}{N} = w_t. \]

- Perfect competition \( \Rightarrow \) price equals marginal cost:

\[ p_x = \frac{1}{A} \left( \frac{r + \delta}{\alpha} \right)^\alpha \left( \frac{w}{1 - \alpha} \right)^{1-\alpha}. \]

- One representative firm \( \rightarrow \) aggregate supply of intermediate good is \( X \).
Household

- Net profit income (from final good producers):
  \[ \Pi_t = \int_{i=0}^{\Omega} \pi_t di - s_t \Phi. \]

- Utility maximization:
  \[
  \max E_0 \sum_{t=0}^{\infty} \beta^t [\log(C_t) + \gamma \log(1 - N_t)], \\
  \text{s.t.} \\
  C_t + K_{t+1} = w_t N_t + (1 + r_t) K_t + \Pi_t.
  \]
General equilibrium

\[ \Phi = \beta E_t \Lambda_{t+1} (\pi_{t+1} + (1 - \theta_{t+1}) \Phi), \]

\[ \Omega_{t+1} = (1 - \theta_t) \Omega_t + s_t, \]

\[ \pi_t = 1 - p_{xt}, \]

\[ \alpha p_{xt} \frac{Y_t}{K_t} = r_t + \delta, \quad (1 - \alpha) p_{xt} \frac{Y_t}{N_t} = w_t \]

\[ w_t C_t^{-1} = \gamma (1 - N_t)^{-1}, \]

\[ C_t^{-1} = \beta E_t C_{t+1}^{-1} (1 + r_{t+1}). \]

\[ C_t + K_{t+1} - (1 - \delta) K_t + s_t \Phi = A_t K_t^\alpha N_t^{1-\alpha}, \]
Equivalence to standard RBC model

- Suppose $\theta = 1$ and no time-to-build.

- Then $V_t = \pi_t = \Phi$. Hence $p_{xt} = 1 - \Phi$ and $s_t = \Omega_t$.

- The aggregate resource constraint becomes
  \[ C_t + K_{t+1} - (1 - \delta)K_t = (1 - \Phi)A_tK_t^\alpha N_t^{1-\alpha}. \]

- The dynamics of this model are the same as those implied by a standard frictionless RBC model (e.g., King, Plosser and Rebelo, 1988).
Impulse responses

- *Calibration.* $\beta = 0.96, \alpha = 0.4, \delta = 0.1, \tilde{N} = 0.2$ (about 35 hours per week). Let $\Phi = 0.1$. The results are not sensitive to these parameter values.

- Assume $\log(\theta_t) = \eta \log(\varepsilon_t)$. In the U.S. (1949-1996), 1% increase in $\varepsilon$ reduces the business failure rate by 6%, hence we set $\eta = -6$.

- The average business failure rate (at annual frequency) for the U.S. economy implies $\bar{\theta} \approx 0.1$. We simulate the model using two alternative values, $\bar{\theta} = \{0.1, 0.25\}$. These values imply a steady-state markup in the range of $1.5 \sim 4\%$. 
Multisector Model

- The production function:
  \[ y = \int_{j=0}^{1} x_j \, dj. \]
  where the price of \( x_j \) is \( p_j \).

- The demand for \( x_j \):
  \[ x_j = \begin{cases} 
  a_j & \text{if } p_j \leq 1 \\
  0 & \text{if } p_j > 1 
  \end{cases} \]
  where \( \langle a_j \rangle \) is the input-output coefficient matrix.
The production function for intermediate good $j$:

$$X_j = AZ_j F(K_j, N_j).$$

Figure 9. Multi-Sector Model.

The gross profit for a final good producer is

$$\pi = y - \int_0^1 a_j p_j dj.$$
The rest of the model’s structure is similar:

\[ V_t = \beta E_t \Lambda_{t+1} (\pi_{t+1} + (1 - \theta_{t+1}) V_{t+1}), \quad \Omega_{t+1} = (1 - \theta_t) \Omega_t + s_t, \]
\[ y = \int_{j=0}^{1} a_j dj = 1, \quad Y_t = \int_{i=0}^{\Omega_t} ydi = \Omega_t, \quad \Pi = \int_{i=0}^{\Omega} \pi di - s \Phi, \]
\[ C_t + K_{t+1} = w_t N_t + (1 + r_t) K_t + \Pi_t, \text{ where } K = \int_{0}^{1} K_j dj \text{ and } N = \int_{0}^{1} N_j dj. \]

The first order conditions for the household are the same as before.

Profit maximization for each intermediate good firm in sector \(j\) gives \(\alpha p_j \frac{X_j}{K_j} = r + \delta\) and \((1 - \alpha) p_j \frac{X_j}{N_j} = w.\) → Marginal cost of good \(j:\)

\[ p_j = \frac{1}{AZ_j} \left( \frac{r + \delta}{\alpha} \right)^\alpha \left( \frac{w}{1 - \alpha} \right)^{1-\alpha}. \]
The aggregate output

\[ Y = \int_0^\Omega \left( \int_0^1 a_j d_j \right) d_i = \int_0^1 (a_j \Omega) d_j, \]

where \( a_j \Omega = X_j \) is the aggregate demand for intermediate good \( j \).

Hence, \( \frac{X_j}{X_i} = \frac{a_j}{a_i}, \frac{Z_j K_j}{a_j} = \frac{Z_i K_i}{a_i} \) and \( \frac{Z_j N_j}{a_j} = \frac{Z_i N_i}{a_i} \).

\[ \rightarrow K_j = \left( \int_0^1 \frac{a_i}{Z_i} d_i \right) \frac{a_j}{Z_j} K, \quad N_j = \left( \int_0^1 \frac{a_i}{Z_i} d_i \right) \frac{a_j}{Z_j} N. \]

Take the normalization, \( \left( \int_0^1 \frac{a_i}{Z_i} d_i \right) = 1 \), we have
\[ K_j = \frac{a_j}{Z_j} K, \]
\[ N_j = \frac{a_j}{Z_j} N. \]

- Substituting \( K_j \) and \( N_j \) into \( X_j = AZ_j K_j^\alpha N_j^{1-\alpha} \) gives

\[ X_j = a_j AK^\alpha N^{1-\alpha}. \]

- In equilibrium the final good production function becomes

\[ Y = \int_{j=0}^{1} (a_j \Omega) dj = \int_{j=0}^{1} X_j dj = AK^\alpha N^{1-\alpha}. \]
Impulse responses

- Impulse responses of aggregate variables, such as \{Y, C, I, N\}, to aggregate technology shocks are the same as before.

- Impulse responses of sectors to aggregate and sector-specific technology shocks:

\[
K_j = \frac{a_j}{Z_j} K,
\]

\[
N_j = \frac{a_j}{Z_j} N.
\]

\[
X_j = a_j Y.
\]

- Equivalence to standard RBC model: Yes, if \( \theta = 1 \) and no time to build.
Explaining Heterogeneity

- Although our model is broadly consistent with stylized facts, it lacks the ability to explain heterogeneous responses across sectors.

- Consider final good firms are heterogenous because each firm $i$ gets a different draw of $a_j$. Namely, firm $i$ can transform one unit of intermediate good $j$ into $a(i,j)$ units of final good. \( \text{Input-output matrix} = \{a(i,j)\}_{i \in [0,\Omega], j \in [0,1]} \).

- The production function:

\[
y_i = \int_0^1 a(i,j) I(i,j) dj,
\]

where $I(i,j) = 1$ if $a(i,j) \geq p_j$ and $I(i,j) = 0$ if $a(i,j) < p_j$. 
Assume \( f_j(a_{i,j}) \neq f_k(a_{i,j}) \) if \( j \neq k \). Denote 
\[ F_j(p_j) = \Pr[a(i,j) \geq p_j] = \int_{p_j} a(i,j)f_j(a)da. \]

The aggregate demand for intermediate good \( j \), by the law of large number, is then 
\[ X_j = \int_0^\Omega I(i,j)di = F_j\Omega. \]

The negative of price elasticity of demand for \( X_j \) is 
\[ \epsilon_j = \frac{p_jf_j(a)}{F_j(p_j)} > 0. \]
**Impulse responses to sector-specific technology shocks.**

- Around the steady state the percentage change of factor demand with respect to $Z_j$ are given by

  $$\hat{K}_j = (\epsilon_j - 1)\hat{Z}_j,$$
  $$\hat{N}_j = (\epsilon_j - 1)\hat{Z}_j;$$

- Hence, allowing for heterogeneity in $f_j(a)$ can explain the heterogeneous responses of inputs across sectors. This has little effects on the impulse responses of the model to aggregate technology shocks.
Responses to Demand.
Discussion

- A micro level rigidity in factor-demand does not by itself imply any aggregate rigidities, as long as $\Omega$ is variable.

- Example 1:

  $$y_i = \int a_{i,j} I(i,j) di,$$

  where $a_{i,j} \sim $ Pareto distribution $F(a) = 1 - \left(\frac{1}{a}\right)^\sigma$. Assume $\theta = 1$ and no time-to-build, we obtain (for $\sigma > 1$)

  $$Y = A(\Phi) \left( \int_0^1 X_j^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}},$$

  $$A(\Phi) \equiv \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma}{\sigma-1}} \left( \frac{1}{\sigma \Phi} \right)^{\frac{1}{\sigma-1}}.$$

- Example 2:
\[ y_i = a_i k + b_i n, \]
where \( k \) is capital, \( n \) is labor, and \( \{a_i, b_i\} \sim \text{Pareto distribution.} \)

- Let the demand functions be
  
  \[ k = \alpha \text{ if } a_i \geq r, \text{ otherwise } k = 0; \]
  
  \[ n = \beta \text{ if } b_i \geq w, \text{ otherwise } n = 0; \]

  where \( \{r, w\} \) stand for prices of capital and labor.

- If \( \theta = 1 \) and no time-to-build, we obtain
  
  \[ Y = A(\Phi) \left[ \alpha \frac{1}{\sigma} K^{\frac{\sigma-1}{\sigma}} + \beta \frac{1}{\sigma} L^{\frac{\sigma-1}{\sigma}} \right] \frac{\sigma}{\sigma-1}. \]
Example 3: If the Pareto distribution is replaced by the Uniform distribution, then
\[
Y = \int_0^1 X_j dj - \left( \frac{\Phi}{2} \right)^{\frac{1}{2}} \left( \int_0^1 X_j^2 dj \right)^{\frac{1}{2}}.
\]

Example 4: Define production function
\[
y_i = \int_0^1 h(a_{i,j}) I(i,j) dj,
\]
where \( h \) is a truncated linear function satisfying
\[
h(a) = \begin{cases} 
a & \text{if } a \leq a_{\text{max}} \\
a_{\text{max}} & \text{if } a > a_{\text{max}}
\end{cases},
\]
where \( a_{\text{max}} \in (1, \infty) \) is an arbitrary truncation point.
• Under Pareto distribution \((\sigma = 1)\), we have

\[
Y = \frac{(1 + \Phi)a_{\text{max}}}{\exp(\Phi)} \exp\left\{ \int_0^1 \log(X_j)d_j \right\},
\]

which is the Cobb-Douglas function with continuum of inputs.

• A special case:

\[
y_i = h(a_i)k + h(b_i)n.
\]

We have

\[
Y = \tilde{B}(\Phi)K^{\frac{\alpha}{\alpha+\beta}} L^{\frac{\beta}{\alpha+\beta}}.
\]
Conclusion

- We have proposed a flexible price RBC model with entry and exit to explain the puzzling effects of technology shocks, especially the asymmetric impacts of aggregate and sector-specific technology shocks on sectorial activity.

- Key elements of our explanation are net business formation at the aggregate level and factor-demand rigidity at the micro-level. Our model collapses to a standard frictionless one-sector RBC model if there is no time-to-build upon firms’ entry.

- Our model provides a micro foundation for various aggregate production functions.

- We view our approach as an alternative to the sticky price approach advocated by Gali (1999) and Basu et al. (2006).